



Problem Solving
for Irish Second
Level
Mathematicians

Problem Solving for Irish Second level Mathematicians

Senior Level

Time allowed: 60 minutes

Rules and Guidelines for Contestants

1. You are **not** allowed to use a calculator or any measuring device (e.g. ruler or protractor).
2. **Use a pencil to fill out the answer sheet.** If you make a mistake, you can erase the error and correct it.
3. Write your name clearly (in block capitals) in the space provided in the answer sheet.
4. You should have some extra sheets of your own paper for rough work while you are doing the questions.
5. When you have decided on your answer for a particular question, carefully mark your choice for that question on the answer sheet.
6. Do not make any other marks on the answer sheet other than to write your name and to mark your answers to the questions.

7. Some of the questions are quite difficult, and we do not expect that many people will have time to think about all of them in 60 minutes. You will probably do better if you concentrate on a few rather than trying to guess the answers to all of the questions. The questions at the beginning are easier than those at the end. The problems are meant to encourage you to think! Don't be in a rush to mark your answer to any of the questions – take your time, read the questions carefully and make sure you understand what is being asked before you start to figure out the answer.

8. **There is no pass/fail mark in PRISM.** Correct answers will score one point each; incorrect or omitted answers will score zero.

Good luck and thank you for participating in PRISM

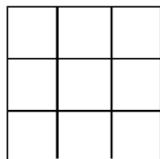
We hope you will enjoy the problems!



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Senior Level 2011

- 1.** How many squares are there in the figure below?



- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14
- 2.** A car travelled from point A to point B at 40 kilometres per hour (km/h) and immediately travelled back from B to A at 60 km/h. What was the average speed of the car for the entire journey?
- (A) 46 km/h (B) 47 km/h (C) 48 km/h (D) 49 km/h (E) none of these
- 3.** Which of the following numbers is the greatest?
- (A) $\frac{5}{7}$ (B) $\frac{3}{4}$ (C) $\frac{7}{9}$ (D) $\frac{4}{5}$ (E) $\frac{9}{11}$
- 4.** In a knock-out chess tournament involving 900 players, how many games must be played to produce a winner? Note: Assume that a game cannot end in a tie (draw). In any round where there are an odd number of players remaining, one player will be given a bye to the next round.
- (A) 798 (B) 800 (C) 899 (D) 900 (E) 1,000
- 5.** How many different selections of five beads can be drawn from a supply of red beads, blue beads and white beads? (Beads are identical apart from colour).
- (A) 18 (B) 15 (C) 12 (D) 3 (E) None of these
- 6.** Which of the following statements is not always true for whole numbers a and b ?
- (A) If both a and b are even, then $a \times b$ is even
(B) If $a \times b$ is even, then both a and b are even
(C) If both a and b are odd, then $a \times b$ is odd
(D) If $a \times b$ is odd, then both a and b are odd
(E) If both a and b are positive, then $a \times b$ is positive.

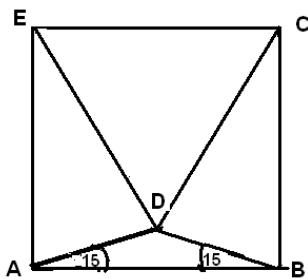
7. Suppose that f is a polynomial such that $f(x^2+1) = x^4+2x^2+1$, what is $f(x+2)$?
- (A) x^4+2x^2+2 (B) x^4+4x+2 (C) x^4+1 (D) $(x+2)^2$ (E) $x+2$

8. How many positive 2-digit whole numbers have either 3 or 7 as a factor?
- (A) 47 (B) 33 (C) 44 (D) 40 (E) 43

9. Some time ago, Ann was three times as old as Bob. Four years later she was twice as old as him. By how many years is Ann older than Bob?

- (A) 4 (B) 3 (C) 8 (D) 12 (E) More information needed.

10. In the figure below, $ABCE$ is a square and the triangle ADB is isosceles with angles DAB and DBA both equal to 15 degrees. Which of the following statements concerning the relative magnitudes of the lengths of the sides of the triangle DEC is true?



- (A) $|ED| = |CD| = |EC|$
(B) $|ED| = |CD| > |EC|$
(C) $|ED| = |CD| < |EC|$
(D) $|ED| > |CD| = |EC|$
(E) $|ED| < |CD| = |EC|$

11. What are all the solutions of the equation $x^2 - 2\sqrt{x^2} - 8 = 0$?

- (A) $x = -2, -4$ (B) $x = 2, 4$ (C) $x = -4, 2, 0, 8$ (D) $x = -8, -2, 2, 4$ (E) $x = -4, 4$

12. The average (arithmetic mean) of 4 numbers is 44. The average of another 5 numbers is 53. What is the average of all 9 numbers grouped together?

- (A) 49 (B) 47.5 (C) 46 (D) 440 (E) 53

13. When the polynomial $f(x) = x^3 - 2x^2 + ax + b$ is divided by $x - 1$, the remainder is 1. When $f(x)$ is divided by $x + 2$, the remainder is -8. What are the values of a and b ?

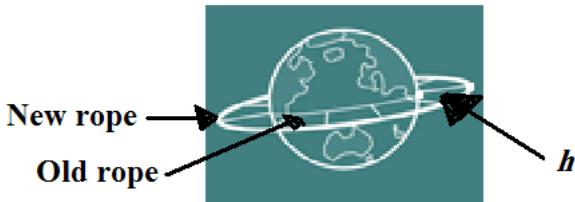
- (A) $a = -2, b = 4$ (B) $a = -2, b = 2$ (C) $a = -1, b = 3$ (D) $a = 1, b = -8$ (E) $a = -8, b = 1$

14. Which of the following shapes has the greatest area?

- (A) An equilateral triangle with sides of length 1
- (B) A square with sides of length 1
- (C) A circle with circumference of length 1
- (D) A circle with radius of length 1.
- (E) A square with diagonal of length 1.

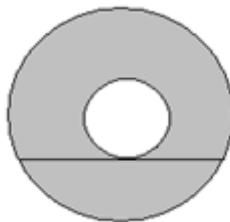
15. Suppose that a rope is wrapped around the Equator of the Earth so that it

touches the Earth at all points. (Thus the circumference of the rope is the same as that of the Earth, which we take as 40,000,000 metres.) Now imagine that we add 1 metre of rope to the existing rope so that we now have a new rope whose length is 40,000,001 metres. This rope will be a little slack at the position in the Equator at which the piece was added. But suppose we spread the new rope around the Equator so that it floats at a uniform distance h from all points on the Equator. In terms of π , what is the value of h in metres?



- (A) $\frac{\pi}{1000}$
- (B) $\frac{\pi}{100}$
- (C) $\frac{\pi}{10}$
- (D) $\frac{1}{\pi}$
- (E) $\frac{1}{2\pi}$

16. In the figure below, the two circles have the same centre. The length of the chord that is tangent to the smaller circle has length 10 cm. What is the area of the ring (the region shaded between the two circles)?



- (A) 25π
- (B) 50π
- (C) 100π
- (D) $25\pi^2$
- (E) $100\pi^2$

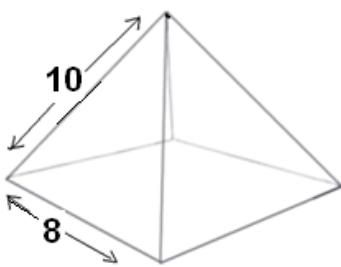
17. The triangle bounded by the lines $y = 0$, $y = x$ and $x = 1 + u$, with u positive, has area equal to 8 m^2 . What is u in metres?

- (A) 3
- (B) 4
- (C) 5
- (D) 6
- (E) 7

18. Three men and six boys can do a job in 2 hours. Two men can do the job in 4.5 hours. How many hours would it take two boys to do the job?

- (A) 6 (B) 9 (C) 12 (D) 18 (E) none of these

19. In the pyramid shown below, each of the four triangular outer surfaces that converge at the top has slant height 10 metres. The base of the pyramid is a square of side 8 metres. What is the volume of the pyramid in cubic metres?



- (A) $\frac{648}{3}\sqrt{68}$ (B) $\frac{648}{7}\sqrt{164}$ (C) 2230.00 (D) $\frac{1648}{5}$ (E) none of these

20. x is a positive integer such that $x+75$ is the square of an integer, and $x+250$ is the square of another integer. If x_1 is the smallest value of x that satisfies the above conditions, and x_2 is the second smallest value of x that satisfies the above conditions, what is the difference $x_2 - x_1$?

- (A) 10 (B) 49 (C) 75 (D) 144 (E) 175

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